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<b>1. REPORT DATE (DD-MM-YYYY)</b>		<b>2. REPORT TYPE</b>		<b>3. DATES COVERED (From - To)</b>	
<b>4. TITLE AND SUBTITLE</b>				<b>5a. CONTRACT NUMBER</b>	
				<b>5b. GRANT NUMBER</b>	
				<b>5c. PROGRAM ELEMENT NUMBER</b>	
<b>6. AUTHOR(S)</b>				<b>5d. PROJECT NUMBER</b>	
				<b>5e. TASK NUMBER</b>	
				<b>5f. WORK UNIT NUMBER</b>	
<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b>				<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b>	
<b>9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b>				<b>10. SPONSOR/MONITOR'S ACRONYM(S)</b>	
				<b>11. SPONSOR/MONITOR'S REPORT NUMBER(S)</b>	
<b>12. DISTRIBUTION / AVAILABILITY STATEMENT</b>					
<b>13. SUPPLEMENTARY NOTES</b>					
<b>14. ABSTRACT</b>					
<b>15. SUBJECT TERMS</b>					
<b>16. SECURITY CLASSIFICATION OF:</b>			<b>17. LIMITATION OF ABSTRACT</b>	<b>18. NUMBER OF PAGES</b>	<b>19a. NAME OF RESPONSIBLE PERSON</b>
<b>a. REPORT</b>	<b>b. ABSTRACT</b>	<b>c. THIS PAGE</b>			<b>19b. TELEPHONE NUMBER (include area code)</b>

# **Final Performance Report**

Grant number: USAF 9550-09-1-0228

Project title: Theory and Applications of Nonlinear Optics in Optically-induced Photonic Lattices

Funding period: 3/15/2009 to 11/30/2011

Reporting period: 3/15/2009 to 11/30/2011

Principal Investigator: Jianke Yang, University of Vermont

Program Manager: Dr. Arje Nachman

## **I. Summary of the P.I.'s activities in this period**

In this funding period, the P.I. investigated nonlinear light propagation in one- and two-dimensional optically-induced photonic lattices both theoretically and experimentally (the theoretical work was performed by the P.I. and his postdocs, while the experiments were performed by the P.I.'s collaborators). Photonic lattices in these works were created by optical induction and were highly tunable in real time, thus they provided a convenient medium to study novel physical phenomena of light propagation in periodic and quasi-periodic media.

In this funding period, the P.I. discovered many new types of nonlinear localized modes (optical solitons) in photonic lattices, such as saddle solitons, truncated-Bloch-mode solitons, two-dimensional embedded solitons, soliton trains, and arbitrary-shape solitons. The finding of these new types of optical solitons offers new possibilities for nonlinear light localization in photonic lattices. His theoretical discovery of arbitrary-shape solitons in photonic lattices also led to a novel image transmission scheme through nonlinear media, and this scheme was successfully demonstrated experimentally in his collaborator's optics lab. In addition, he demonstrated both theoretically and experimentally that a nonlinear beam can be reflected by a negative (repulsive) defect in a photonic lattice if the incident angle is below a threshold value. Above this threshold angle, the beam simply passes through the defect. This phenomenon provides a way to use the incident angle to control beam propagation in a lattice network. Furthermore, the P.I. demonstrated that a negative defect can guide various linear defect modes (such as vortex modes) without any diffraction. Besides these physical investigations, the P.I. also developed a mathematical technique (the exponential asymptotics method) for the study of existence and linear stability of solitons in photonic lattices. This sophisticated technique makes analytical investigation of nonlinear phenomena in photonic lattices possible for the first time. In addition, the P.I. invented a new numerical method --- the Newton-conjugate-gradient method, which can efficiently determine solitons in photonic lattices. This algorithm is orders of magnitude faster than the previous ones, thus it is a powerful tool for numerical studies of nonlinear optics problems.

During this period, the P.I. published two monographs, one invited book chapter, and 11 refereed journal articles (as well as a number of conference proceedings articles). The two monographs were published by the prestigious publishers Cambridge University Press and SIAM. All journal articles appeared in the leading optics and applied mathematics journals such as Optics Letters, Physical Review A, Physica D, Studies in Applied Mathematics, Proceedings of Royal Society A, Journal of Computational Physics, etc.

In this funding period, the P.I. collaborated with the following colleagues/postdocs:

Zhigang Chen (San Francisco State University)  
Triantaphyllos Akylas (MIT)  
Yuri Kivshar (Australian National University)  
Maksim Skorobogatiy (Ecole Polytechnique de Montréal, Canada)  
Jiandong Wang (University of Vermont, postdoc)  
Guenbo Hwang (University of Vermont, postdoc)  
Sean Nixon (University of Vermont, postdoc)

Regarding his other activities, he delivered a two-hour tutorial lecture in the sixth IMACS international conference on nonlinear evolution equations and wave phenomena (Athens, Georgia, USA, 2009). He also co-chaired the second international conference on nonlinear waves (Beijing, China, June 26-29, 2010). In addition, he attended five research conferences on optics and applied mathematics and presented talks.

## **II. Summary of the P.I.'s research results in this period**

### **(1) Theory and application of nonlinear light phenomena in photonic lattices (6 papers)**

In this project, the P.I. extensively investigated nonlinear light propagation in photonic lattices both theoretically and experimentally. He demonstrated that due to bandgap structures of photonic lattices, a number of novel optical solitons arise which have no counterpart in homogeneous media. One example is saddle solitons which arise due to simultaneous compensation of normal and anomalous (saddle-shaped) diffractions with self-focusing and self-defocusing hybrid nonlinearity in ionic-type photonic lattices (Opt. Lett. 2009). The phase and spectrum structures of these saddle solitons are different from all previously reported optical solitons. Another example is truncated-Bloch-mode solitons which form by truncation of Bloch modes under nonlinear effects (PRA 2009). The third example is two-dimensional embedded solitons in waveguide arrays (PRA 2010). These solitons exist inside Bloch bands of the waveguide, and their discovery is very surprising since intuitively solitons are only expected outside Bloch bands. The fourth example is solitons of arbitrary shapes such as H shape and cross shape (Opt. Lett. 2011). One more example is soliton trains which are stable against transverse

perturbations (PRA 2011). The finding of these new types of optical solitons offers new possibilities for nonlinear light localization in photonic lattices.

In this project, the P.I. also made important progress on physical applications of nonlinear photonic-lattice research. One application was that, based on the theoretical demonstration of stable arbitrary-shape solitons in photonic lattices, the P.I. and collaborators proposed a novel image transmission scheme through nonlinear media (Opt. Lett. 2011). In this scheme, texts and images are sent as solitons of various shapes. This scheme was successfully demonstrated experimentally in the P.I.'s collaborator's lab. Another application was that the P.I. proposed a new way to control the direction of beam propagation in photonic-lattice networks (PRA 2011). This new control mechanism was based on the P.I.'s theoretical demonstration that in a photonic lattice with a local negative defect, a nonlinear beam is reflected if the incident angle is below a threshold value. Above this threshold angle, the beam simply passes through the defect. This phenomenon then provides a way to use the incident angle to control beam propagation in a lattice network. This mechanism was tested experimentally and found to work very well.

(2) Linear light trapping in photonic lattices with structured defects (2 papers)

Linear light trapping by structured defects in photonic lattices is an interesting physical phenomenon. The creation and maintenance of a local defect in a photonic lattice by optical induction is not easy, but the P.I. and his experimental collaborator have succeeded in doing so by ingenious spectral filtering combined with other techniques. With a local defect successfully created, the P.I. then demonstrated that even though the refractive-index contrast of optically-induced photonic lattices is small (on the order of  $10^{-3}$ ), a negative (repulsive) defect can still guide various defect modes without any diffraction (book chapter, 2009). Indeed, we even observed a linear vortex mode guided by a localized defect (Opt. Lett. 2010).

(3) Development of the exponential asymptotics method for the existence and stability of solitons in photonic lattices (3 papers)

Most theoretical photonic-lattice research in the literature was performed by numerical computations since analytical studies were difficult. But analytical investigations are very important for a deep understanding of nonlinear phenomena in photonic lattices. In this project, the P.I. and his postdocs/collaborators made important progress on the analytical treatment of existence and stability of solitons in photonic lattices. Specifically, we developed a sophisticated exponential asymptotics method which allowed us to analytically determine the locations of solitons in photonic lattices as well as which of those solitons are linearly stable (Physica D 2011, Stud. Appl. Math. 2011). This method was applied successfully to both linear and nonlinear lattices. Extension of these works also enabled us to

analytically determine the existence and locations of multi-soliton bound states in photonic lattices (Proc. Roy. Soc. A 2012). The analysis we developed not only advanced photonic lattice research, but also advanced methodologies for applied mathematics.

- (4) Development of Newton-conjugate-gradient methods for solitary wave computations (one paper)

In this project, the P.I. also invented a new numerical method --- Newton-conjugate-gradient method, which can efficiently determine optical solitons in photonic lattices (J. Comp. Phys. 2009). The idea of this method is the following. Let us consider the computation of solitons in a general nonlinear system

$$L_0 u = 0, \quad (1)$$

where  $L_0$  is a nonlinear operator and  $u(x)$  is a soliton solution. To compute this soliton, we first use the Newton's iteration,

$$u_{n+1} = u_n + \Delta u_n,$$

where  $\Delta u_n$  is obtained from the linear Newton-correction equation

$$L_1 \Delta u_n = -L_0 u_n, \quad (2)$$

and  $L_1$  is the linearization operator of Eq. (1). Traditional Newton's method would discretize Eq. (2) into a matrix system and then solve it by LU decomposition. Our idea is to solve (2) by preconditioned conjugate-gradient iterations. Our numerical testing shows that this new algorithm is orders of magnitude faster than the existing ones, and is thus a powerful tool for numerical studies of nonlinear optics problems.

### III. List of publications in this period

#### Monographs:

1. J. Yang, *Nonlinear Waves in Integrable and Nonintegrable Systems* (SIAM, Philadelphia, 2010).
2. M. Skorobogatiy and J. Yang, *Fundamentals of Photonic Crystal Guiding* (Cambridge University Press, Cambridge, UK, 2009).

#### Book chapters:

- (Invited) J. Yang, X. Wang, J. Wang and Z. Chen, "Light localization by defects in optically induced photonic structures", chapter in "Nonlinearities in Periodic Structures and Metamaterials", C. Denz, S. Flach and Y.S. Kivshar, ed. (Springer, Berlin, 2009), pp 127-143.

#### Journal articles:

1. T.R. Akylas, G. Hwang and J. Yang, "From nonlocal gap solitary waves to bound states in periodic media", *Proc. Roy. Soc. A* 468, 116-135 (2012).
2. G. Hwang, T.R. Akylas and J. Yang, "Solitary waves and their linear stability in nonlinear lattices", *Stud. Appl. Math.* DOI: 10.1111/j.1467-9590.2011.00538.x (published online on Nov. 15, 2011).
3. J. Yang, "Transversely stable soliton trains in photonic lattices", *Phys. Rev. A* 84, 033840 (2011).
4. G. Hwang, T.R. Akylas and J. Yang, "Gap solitons and their linear stability in one-dimensional periodic media", *Physica D* 240, 1055–1068 (2011).
5. J. Wang, Z. Ye, A. Miller, Y. Hu, C. Lou, P. Zhang, Z. Chen and J. Yang, "Nonlinear beam deflection in photonic lattices with negative defects", *Phys. Rev. A* 83, 033836 (2011).
6. J. Yang, P. Zhang, M. Yoshihara, Y. Hu, and Z. Chen, "Image transmission using stable solitons of arbitrary shapes in photonic lattices", *Opt. Lett.* 36, 772–774 (2011).
7. J. Yang, "Fully localized two-dimensional embedded solitons", *Phys. Rev. A* 82, 053828 (2010).
8. D. Song, X. Wang, D. Shul'dman, J. Wang, L. Tang, C. Lou, J. Xu, J. Yang, and Z. Chen, "Observation of bandgap guidance of optical vortices in a tunable negative defect", *Opt. Lett.* 35, 2106 (2010).
9. Y. Hu, C. Lou, P. Zhang, J. Xu, J. Yang, and Z. Chen, "Saddle solitons: A new balance between bi-diffraction and hybrid nonlinearity", *Opt. Lett.* 34, 3259 (2009) [reprinted in *Virtual J. Ultrafast Science*, Dec, 2009].
10. J. Wang, J. Yang, T.J. Alexander, and Y.S. Kivshar, "Truncated-Bloch-wave solitons in optical lattices", *Phys. Rev. A* 79, 043610 (2009).
11. J. Yang, "Newton-Conjugate-Gradient Methods for Solitary Wave Computations", *J. Comp. Phys.* 228, 7007–7024 (2009).